COMBINED HEAT TRANSFER UNDER CONDITIONS OF FREE CONVECTION

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On the basis of the boundary-layer equations solutions are obtained for the problems of convective and combined heat transfer under conditions of free convection in transparent and gray media. The solution for convective heat transfer in a transparent medium is obtained by means of a Taylor expansion of the temperature function. Expressions for the radiative and convective components of combined heat transfer are also presented, and it is shown that these fluxes are interrelated quantities.

The nonisothermal motion of a medium along a wall is accompanied by formation of hydrodynamic and thermal boundary layers of thickness $\delta = \varphi(x)$ and $\delta_T = \psi(x)$, respectively (Fig. 1).

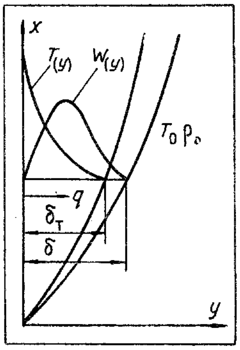

Fig. 1. Model of boundary layer near wall.

The relationship between the boundary layers can be represented in the form $\delta_T/\delta = \xi$.

The momentum and energy transfer near a vertical wall can be represented by the following system of equations:

$$\frac{\partial(\rho W_x)}{\partial x} + \frac{\partial(\rho W_y)}{\partial y} = 0, \tag{1}$$

$$\rho W_x \frac{\partial W_x}{\partial x} + \rho W_y \frac{\partial W_x}{\partial y} = \mu \frac{\partial^2 W_x}{\partial y^2} + g(\rho - \rho_0), \tag{2}$$

$$\rho c_p W_x \frac{\partial T}{\partial x} + \rho c_p W_y \frac{\partial T}{\partial y} = \lambda \frac{\partial^2 T}{\partial y^2} + \frac{\sigma_0}{k} \frac{\partial^2 T^4}{\partial y^2}, \tag{3}$$

$$P = \rho RT. \tag{4}$$

The second term on the right-hand side of the energy transfer equation (3) characterizes the change in the radiant flux, which for media with considerable optical density $k\delta > 6$ is represented using the diffusional concept of energy transfer.

The second term on the right-hand side of the momentum equation (2), which takes the effect of gravitational forces into account, can be represented in the somewhat different form:

$$g(\rho-\rho_0)=\frac{gP}{R}\left(\frac{1}{T}-\frac{1}{T_0}\right),$$

where T_0 is the temperature of the medium outside the boundary layer. Expanding the function $1/T$ in a Taylor series near the point $1/T_0$ and cutting off the series after three terms, we obtain

$$g(\rho-\rho_0)=\frac{gP}{RT_0}\left(2-\frac{3T}{T_0}+\frac{T^2}{T_0^2}\right).$$

To solve the system of equations (1)–(4) it is necessary to take the following boundary conditions into account:

$$W_x|_{y=0}=0;\quad W_y|_{y=\delta}=0;\quad \left.\frac{\partial W_x}{\partial y}\right|_{y=\delta}=0;$$
$$T|_{y=0}=T_w;\quad T|_{y=\delta_T}=T_0;\quad \left.\frac{\partial T}{\partial y}\right|_{y=\delta_T}=0;$$
$$W_y|_{y=0}=0.$$

We introduce the new variables $\eta=y/\delta$, $\eta_T=y/\delta_T$.

The velocity field can be represented in the following form:

$$\frac{\rho W_x}{\rho_0 u}=\varphi(\eta), \tag{5}$$

where u is a constant with the dimension of velocity.

Differentiating both sides of (5) with respect to x, we obtain

$$\frac{\partial(\rho W_x)}{\partial x}=-\rho_0 u\varphi'(\eta)\eta\frac{1}{\delta}\frac{\partial\delta}{\partial x}.$$

Using the continuity equation (1), we find the y-component of the velocity

$$\rho W_y=\rho_0 u\frac{d\delta}{dx}\left[\varphi(\eta)\eta-\int_0^{\eta}\varphi(\eta)d\eta\right].$$

The temperature field can be represented in the following form:

$$\frac{T}{T_w-T_0}=\psi(\eta_T). \tag{6}$$

Differentiating both sides of (6) with respect to x and y, we obtain

$$\frac{\partial T}{\partial x}=-(T_w-T_0)\psi'(\xi\eta)\frac{1}{\delta}\frac{\partial\delta}{\partial x},$$
$$\frac{\partial T}{\partial y}=(T_w-T_0)\frac{\partial\psi}{\partial(\xi\eta)}\frac{1}{\delta}.$$

Thus, using the above transformations, we reduce the momentum equation (2) and the energy transfer equation (3) to the form

$$-(\varphi(\eta)\psi(\eta_T))'\int_0^{\eta}\varphi(\eta)d\eta=A_1[\varphi(\eta)\psi(\eta_T)]''+$$
$$+A_3\left[2\frac{T_0^2}{\Delta T}-3\frac{T_0}{\Delta T}\psi+\psi^2\right], \tag{7}$$

$$-\psi'(\xi\eta)\int\limits_0^{\xi\eta}\varphi(\xi\eta)\,d(\xi\eta)=A_2\frac{\partial^2\psi}{\partial(\xi\eta)^2}+A_4\frac{\partial^2(\psi^4)}{\partial(\xi\eta)^2},\tag{8}$$

where

$$A_1=\frac{\nu}{u\delta\dfrac{d\delta}{dx}};\quad A_2=\frac{a}{u\delta\dfrac{d\delta}{dx}};$$

$$A_3=\frac{g\beta\Delta T\delta}{u^2\dfrac{d\delta}{dx}};\quad A_4=\frac{\sigma_0\Delta T^3}{\rho_0 c_p k u\delta\dfrac{d\delta}{dx}}.$$

We find the solution of Eqs. (7) and (8) in series form:

$$\varphi(\eta)=a_0+a_1\eta+a_2\eta^2+a_3\eta^3+a_4\eta^4+\cdots\tag{9}$$

$$\psi(\eta_T)=b_0+b_1(\xi\eta)+b_2(\xi\eta)^2+b_3(\xi\eta)^3+b_4(\xi\eta)^4+\cdots\tag{10}$$

By using the boundary conditions at y = 0 we can find the value of the coefficients a_0 and b_0:

$$a_0=0;\quad b_0=\frac{T_w}{T_w-T_0}.$$

Using Eqs. (9) and (10) in solving the system of equations (7) and (8), we can obtain the values of the coefficients a_2, a_3, a_4, ..., b_2, b_3, b_4, ... in the form of functions of a_1 and b_1.

An analysis of the coefficients b_i obtained enables us to describe the temperature field in the boundary layer by means of the following expression:

$$\frac{T}{T_w-T_0}-\frac{T_w}{T_w-T_0}=b_1\xi\eta-\left[\frac{a_1b_1\xi^4}{24\left[A_2+4A_4\left(\dfrac{T_w}{\Delta T}\right)^3\right]}+\right.$$
$$+\frac{1080\,(b_1\xi)^4}{\left[\dfrac{A_2}{A_4}\left(\dfrac{\Delta T}{T_w}\right)^2+4\dfrac{T_w}{\Delta T}\right]^3}-$$
$$\left.-\frac{120\,(b_1\xi)^4}{\left[\dfrac{A_2}{A_4}+4\left(\dfrac{T_w}{\Delta T}\right)^3\right]^2\left(\dfrac{\Delta T}{T_w}\right)^3}\right]\eta^4.\tag{11}$$

Using the boundary conditions at $\eta = 1$

$$\frac{T-T_w}{T_w-T_0}=-1\ \text{and}\ \frac{dT}{d\eta}=0$$

we find

$$b_1\xi=-\frac{4}{3},\tag{12}$$

$$\xi=\left\{A_4\left\{6\left[\frac{A_2}{A_4}+4\left(\frac{T_w}{\Delta T}\right)^3\right]^2\left(\frac{\Delta T}{T_w}\right)^6+\right.\right.$$
$$\left.+61440-6827\left[\frac{A_2}{A_4}+4\left(\frac{T_w}{\Delta T}\right)^3\right]\left(\frac{\Delta T}{T_w}\right)^3\right\}\times$$
$$\left.\times\left\{a_1\left[\frac{A_2}{A_4}+4\left(\frac{T_w}{\Delta T}\right)^3\right]^2\left(\frac{\Delta T}{T_w}\right)^6\right\}^{-1}\right\}^{1/3}.\tag{13}$$

Substituting the values of the coefficients a_i in Eq. (9) and using the boundary conditions at $\eta = 1$ in the form $W_x = 0$ and $\partial W_x/\partial\eta = 0$, we obtain the following system of equations:

$$a_1\left[1+\frac{4}{3}\frac{\Delta T}{T_w}+\frac{16}{9}\left(\frac{\Delta T}{T_w}\right)^2+\frac{64}{27}\left(\frac{\Delta T}{T_w}\right)^3\right]-\frac{a_1^2}{24A_1}+$$
$$+a_1\left[\frac{32}{3\left[\frac{A_2}{A_4}+4\left(\frac{T_w}{\Delta T}\right)^3\right]\frac{\Delta T}{T_w}}+\right.$$
$$+\frac{512}{3\left[\left(\frac{A_2}{A_4}\right)^2\left(\frac{\Delta T}{T_w}\right)^3+8\frac{A_2}{A_4}+16\left(\frac{T_w}{\Delta T}\right)^3\right]}+$$
$$\left.+\frac{512}{27\left[\frac{A_2}{A_4}+4\left(\frac{T_w}{\Delta T}\right)^3\right]}\right]-\frac{A_3}{A_1}\left\{\frac{\nabla}{2}\frac{\Delta T}{T_w}+\right.$$
$$+\frac{4}{3}\left(\frac{T_0^2}{T_w^2}-\frac{T_0}{T_w}+\frac{1}{6}\right)+$$
$$\left.+\frac{16}{9}\left[\left(\frac{\nabla}{2}\right)\left(\frac{\Delta T}{T_w}\right)^3+\frac{\chi}{6}\left(\frac{\Delta T}{T_w}\right)^2+\frac{\Delta T}{12T_w}\right]\right\}-$$
$$-\frac{16A_3\left[6\left(\frac{T_0}{T_w}\right)^2-7.5\frac{T_0}{T_w}+2\right]\left(\frac{T_w}{\Delta T}\right)^2}{9A_1\left[\frac{A_2}{A_4}+4\left(\frac{T_w}{\Delta T}\right)^3\right]}=0, \tag{14}$$

$$a_1\left[1+\frac{8}{3}\frac{\Delta T}{T_w}+\frac{16}{3}\left(\frac{\Delta T}{T_w}\right)^2+\frac{256}{27}\left(\frac{\Delta T}{T_w}\right)^3\right]+$$
$$+a_1\left[\frac{32}{\left[\frac{A_2}{A_4}+4\left(\frac{T_w}{\Delta T}\right)^3\right]\frac{\Delta T}{T_w}}+\right.$$
$$+\frac{2048}{3\left[\frac{A_2}{A_4}+4\left(\frac{T_w}{\Delta T}\right)^3\right]^2\left(\frac{\Delta T}{T_w}\right)^3}+$$
$$\left.+\frac{2048}{27\left[\frac{A_2}{A_4}+4\left(\frac{T_w}{\Delta T}\right)^3\right]}\right]-$$
$$-\frac{A_3}{A_1}\left\{\nabla\frac{\Delta T}{T_w}+4\left(\frac{T_0^2}{T_w^2}-\frac{T_0}{T_w}+\frac{1}{6}\right)+\right.$$
$$\left.+\frac{64}{9}\left[\frac{\nabla}{2}\left(\frac{\Delta T}{T_w}\right)^3+\frac{\chi}{6}\left(\frac{\Delta T}{T_w}\right)^2+\frac{\Delta T}{12T_w}\right]\right\}-$$
$$-\frac{64}{9}\frac{A_3}{A_1}\frac{\left[6\left(\frac{T_0}{T_w}\right)^2-7.5\frac{T_0}{T_w}+2\right]\left(\frac{T_w}{\Delta T}\right)^2}{\left[\frac{A_2}{A_4}+4\left(\frac{T_w}{\Delta T}\right)^3\right]}-$$
$$-\frac{a_1^2}{6A_1}=0, \tag{15}$$

where

$$\nabla = 2\frac{T_0^2}{\Delta T^2} - 3\frac{T_0 T_{er}}{\Delta T^2} + \frac{T_w^2}{\Delta T^2};$$

$$\chi = 3\frac{T_0}{\Delta T} - 2\frac{T_w}{\Delta T}.$$

It is not possible to find the values of the coefficients a_1 and b_1 in general form, since the system of equations contains two variables $\Delta T/T_W$ and $N = A_2/A_4 = \lambda k/\sigma_0 \Delta T^3$. Therefore, the subsequent solution was obtained for specifically selected values of $\Delta T/T_W$ and N. Altogether we examined 16 variants with four values of $\Delta T/T_W$ [0.1; 0.2; 0.3; 0.4] and four values of the parameter N [0.1; 1; 10; 100], characterizing the ratio of the quantity of heat transferred by heat conduction to the quantity of heat transferred by radiation.

In examining the problem of energy transfer by free convection only, without allowance for radiative transfer, we analytically determined an expression for the convective heat-transfer coefficient

$$\mathrm{Nu} = C\sqrt[3]{\mathrm{Pr}}\sqrt[4]{\mathrm{Gr}}. \tag{16}$$

In this case the coefficient C depends only on the ratio $\Delta T/T_W$ (Table 1). If the temperature drop $(T_W - T_0)$ is inconsiderable, the coefficient C does not change and may be taken equal to 0.315.

Table. 1. The Coefficient C as a Function of the Parameter $\Delta T/T_W$

$\frac{\Delta T}{T_w}$	0.1	0.2	0.3	0.4	0.5
C	0.315	0.317	0.315	0.296	0.277

The expression obtained is in good agreement with Herman's solution obtained by introducing a special stream function.

As distinct from Herman's solution, which substitutes $1/T_0$ for the temperature function $1/T$ in the momentum equation (2), our solution employs the first three terms of the Taylor expansion of the function.

As pointed out above, the solution of the problem in the presence of combined heat transfer was obtained separately for each of the selected variants (of N and $\Delta T/T_W$). As a result of the solution we found an expression that enabled us to determine the convective heat-transfer coefficient under conditions of combined heat transfer:

$$\mathrm{Nu}^* = C^*\sqrt[3]{\mathrm{Pr}}\sqrt[4]{\mathrm{Gr}}, \tag{17}$$

where

$$C^* = (N^{0.118+0.00111N} - 0.57)\frac{\Delta T}{T_w}. \tag{18}$$

The heat flux q_c is given by the expression

$$q_c = \frac{\lambda}{x}\Delta T\,\mathrm{Nu}^*. \tag{19}$$

Thus, for each specific value of the parameters of the medium and the wall it is possible to find the convective heat flux.

In Fig. 2 the convective heat flux is shown as a function of the parameters N and the optical density of the medium $(k\delta)$, in this case CO_2.

An analysis of graph shows that as the optical density of the medium increases the convective component also increases.

As the parameter N decreases, a marked increase in the convective heat flux is also observed. This increase in q_c is attributable to an increase in the temperature difference between the medium and the wall (ΔT), which in turn leads to a decrease in the parameter $N = k\lambda/\sigma_0 \Delta T^3$.

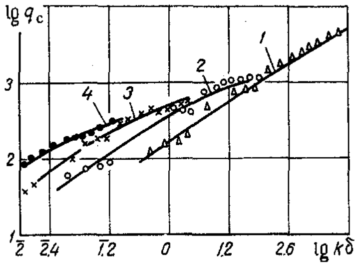

Fig. 2. Convective component of combined heat transfer as a function of the parameters N and kδ: 1—N = 100; 2—100; 3—1; 4—0.1.

As a result of our solution we were able to construct the temperature fields near the wall for various values of the parameter N and $\Delta T/T_w$.

In Fig. 3 the temperature fields are represented for four values of the parameter N and two values of $\Delta T/T_w$ at Pr = 1.

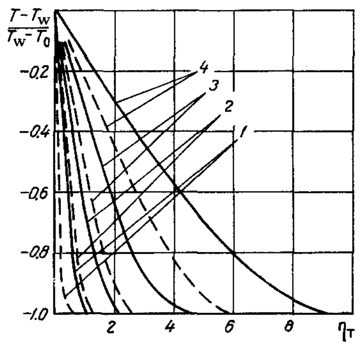

Fig. 3. Temperature field of medium near wall as a function of the parameters N and $\Delta T/T_w$: 1—N = 100; 2—10; 3—1; 4—01; solid line—$\Delta T/T_w$ = 0.2; broken line—$\Delta T/T_w$ = = 0.3.

An analysis of the graphs shows the effect of radiative energy transfer on the formation of the temperature field near a hot wall.

A comparison of the temperature fields for convective heat transfer only and combined heat transfer shows that the temperature gradient near the wall varies with the optical density of the medium. As the optical density increases, the thickness of the boundary layer and the temperature gradient decrease. Consequently, the photons penetrate to a lesser depth without interacting, which causes a decrease in radiative and an increase in convective transfer.

To determine the radiative component we use the expression [2]

$$q_r = \frac{\sigma_0 [T_w^4 - T_\delta]}{\dfrac{1}{A_w} - \dfrac{1}{2}}. \tag{20}$$

Here, it was assumed that $A_w = 0.85$; T_δ is the temperature of the medium at a distance from the wall equal to the photon mean free path l_p.

Since $l_f = 1/k$, we find the value of the temperature T_δ using expression (11) with $\eta_T = 1/k\delta$.

With a certain degree of accuracy the value of the temperature T_δ can be represented in the following way:

$$T_\delta^4 = T_w^4 - T_w^4 \left\{4\frac{\Delta T}{T_w} n \frac{1}{k\delta} - 6\left(\frac{\Delta T}{T_w}\right)^2 n^2 \frac{1}{(k\delta)^2} + 4\left(\frac{\Delta T}{T_w}\right)^3 n^3 \frac{1}{(k\delta)^3} - \left[4\frac{\Delta T}{T_w} m + \left(\frac{\Delta T}{T_w}\right)^4 n^4\right] \frac{1}{(k\delta)^4}\right\}. \quad (21)$$

Values of the coefficients of the temperature field in (21) vary with $\Delta T/T_w$ and N (Table 2).

Table 2. Values of the Coefficients of the Temperature Field n and m as Functions of the Parameters $\Delta T/T_w$ and N

$\frac{\Delta T}{T_w}$	$N=0.1$		$N=1.0$		$N=10$		$N=100$	
	n	m	n	m	n	m	n	m
0.1	0.053642	0.00000088	0.1142	0.00001797	0.2487	0.0004023	0.5353	0.00867
0.2	0.1452	0.00004687	0.3	0.0008583	0.6665	0.02039	1.4256	0.5294
0.3	0.229	0.0003725	0.5187	0.007626	1.1796	0.2196	3.1364	10.009
0.4	0.3499	0.001559	0.7552	0.0349	1.9179	1.7094	5.4855	95.98

All the coefficients were found for each of the above-mentioned variants.

Thus, to find the temperature T_δ and the radiant heat flux q_r it is necessary to consider the effect on them of the parameters N and $\Delta T/T_w$, as well as the Bouguer number $(k\delta)$.

Solutions were obtained for 64 different variants of the above-mentioned parameters for carbon dioxide. The results of the solution for the radiant heat flux are presented in Fig. 4. An analysis of the graph shows that as the optical density of the medium increases the radiative component decreases.

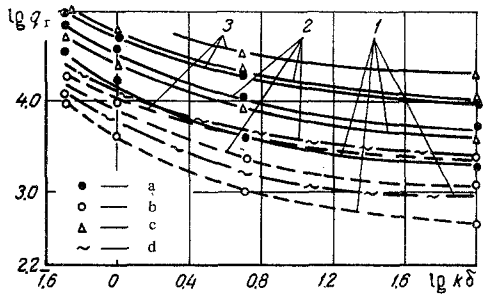

Fig. 4. Radiative components of combined heat transfer as a function of the parameters N, $k\delta$, and $\Delta T/T_w$: 1–$\Delta T/T_w$ = 0.2; 2–0.3; 3–0.4; at N = 1 T_w = 1000° K (a) and 700° K (b), at N = 10 T_w = 1000° K (c) and 700° K (d).

Our theoretical solution of the problem of combined heat transfer under conditions of free convections shows that the processes of thermal energy transfer by convection and radiation are interrelated.

NOTATION

$\delta(x)$ and $\delta_T(x)$ are the thicknesses of the hydrodynamic and thermal layers; ξ is the relation between these layers; x and y are coordinates; W_x and W_y are velocities; $\eta = y/\delta$ and $\eta_T = y/\delta_T$ are dimensionless variables; u is a constant with the dimension of velocity; μ is the dynamic viscosity; g is the acceleration of gravity; T is the temperature of the medium within the boundary layer; T_0 is the temperature of the medium outside of the boundary layer; T is the temperature of the radiative-equilibrium layer; n and m are temperature field coefficients; ρ and ρ_0 are the densities of the medium at T and T_0, respectively; c_p is the specific heat; σ_0 is Boltzmann's constant; k is the absorption coefficient; R and P are the gas constant and the pressure; $\Delta T/T_w$ is the temperature difference ratio; $(T_w - T)$ to T_w; $N = \lambda(k/\sigma)\Delta T^3$ is the parameter characterizing the ratio of the quantity of heat transferred by conduction to the heat transferred by radiation; Bu = $k\delta$ is the Bouguer number; Nu is the Nusselt number for convective heat transfer of a transparent medium; Nu* is the Nusselt number for the convective component of the combined heat transfer; Gr and Pr are the Grashof and Prandtl numbers.

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